Horn Optimization Project

- Generate dataset of various horn design/shapes
- Bezier curves will define shape with a couple varying control points per design

Sample Testing Horn Design/Shapes

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Frequency Sweep from 1000Hz to 10kHz for each design

Determine Directivity and Impedance Values

↓

Create metamodel (Kriging and Polynomial) to represent data

Find optimal solution on Pareto frontier w/ objective functions of…

- Constant linear directivity across frequency sweep
- Minimizing imaginary normal impedance radiation

Find design with desired tradeoff between the two objective functions

1. FEM Helmholtz Simulation

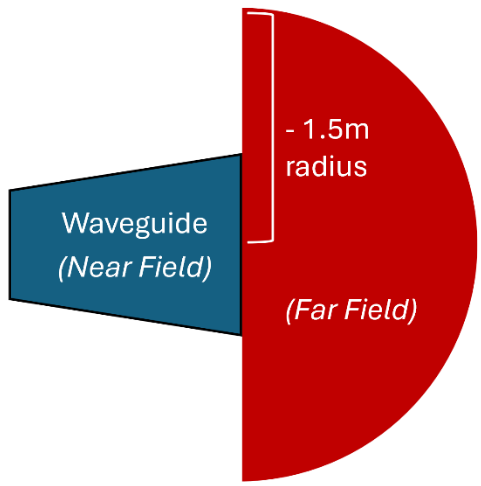
a. Acoustic Parameters

i. Speed of Sound = 343 m/s

ii. Wave Number (k) = (2*pi*frequency) / (Speed of Sound)

b. Define Geometry

i. Set geometry bounds for the horn for near-field calculations, and a semicircle for far-field calculations.



c. Mesh Generation

i. 6 elements per wavelength

ii. Generation for both near and far fields

d. Frequency Domain Helmholtz PDE

i. Helmholtz equation in 2D (time-harmonic)

$$\nabla^2 u + k^2 u = 0$$

ii. Written as...

$$-\nabla \cdot (c \nabla u) + au = f$$

- c = 1 (diffusion coefficient)
- a = -k^2
- f = 0
- m = 0, d = 0 (steady state, no mass or damping terms)

iii. In MATLAB code

```
specifyCoefficients(model, ...
    'm', 0,          ... % no time-dependent mass
    'd', 0,          ... % no damping
    'c', 1,          ... % Laplacian operator (∇²u)
    'a', -k^2,       ... % Helmholtz potential term
    'f', 0);         ... % no source
```

e. Excitation

i. Parameters

- Air Density (rho) = 1.21 kg/m^3
- Omega = 2 * pi * frequency
- Uniform Velocity (v) = 1 m/s

ii. Boundary Condition

- Neumann excitation via velocity source (v = 1 m/s)
- Applied at throat edge

$$\left.\frac{\partial p}{\partial n}\right|_{\text{throat}} = -\rho \cdot \omega \cdot i \cdot v$$

- Integral represent "q" or the flux within the code

f. Wall Boundary Conditions

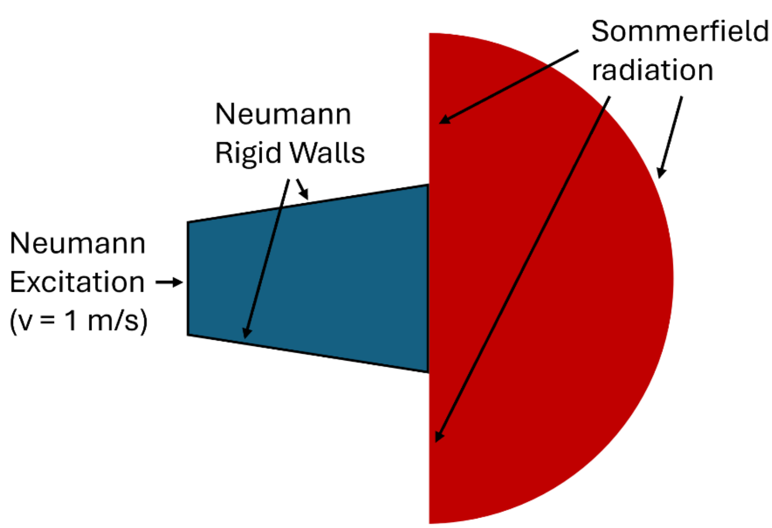
i. Ridid Walls: Neuman with g = 0, enforcing hard boundaries with no normal velocity

$$\left.\frac{\partial p}{\partial n}\right|_{\text{wall}} = 0$$

ii. Semi-Circle Edge: Apples Sommerfield radiation condition to absorb outgoing waves (simulating an infinite domain)

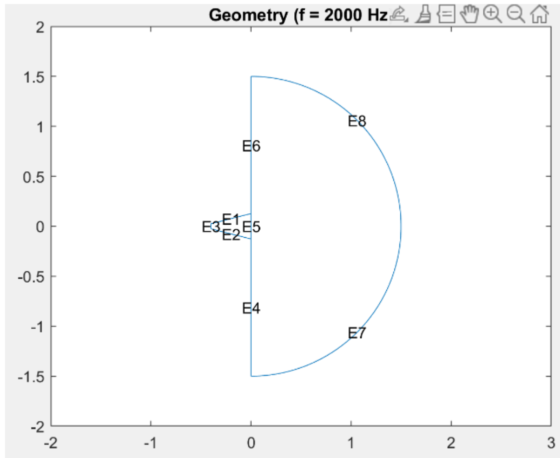
$$\frac{\partial p}{\partial n} = ikp$$

- Despite measurements taking place 1m away the far-field domain is 1.5m in distance to potentially avoid reflections off boundary walls

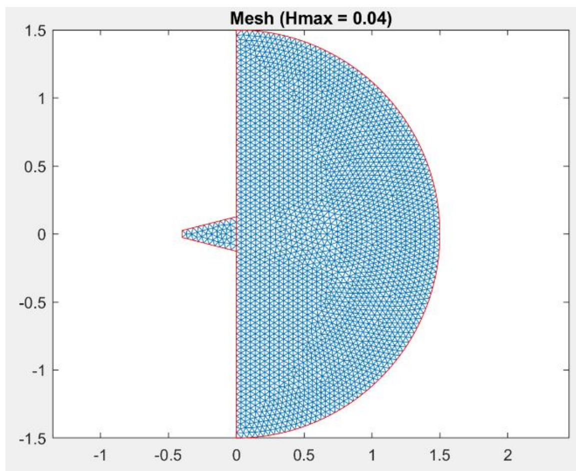


g. Example Simulation Results (2000Hz)

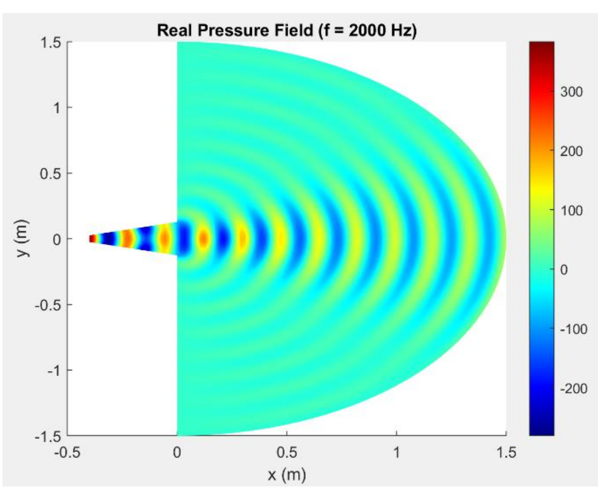
i. Geometry Setup

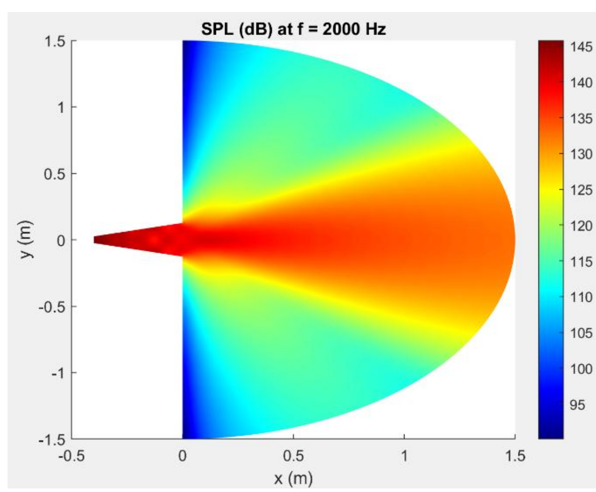


ii. Mesh Generation



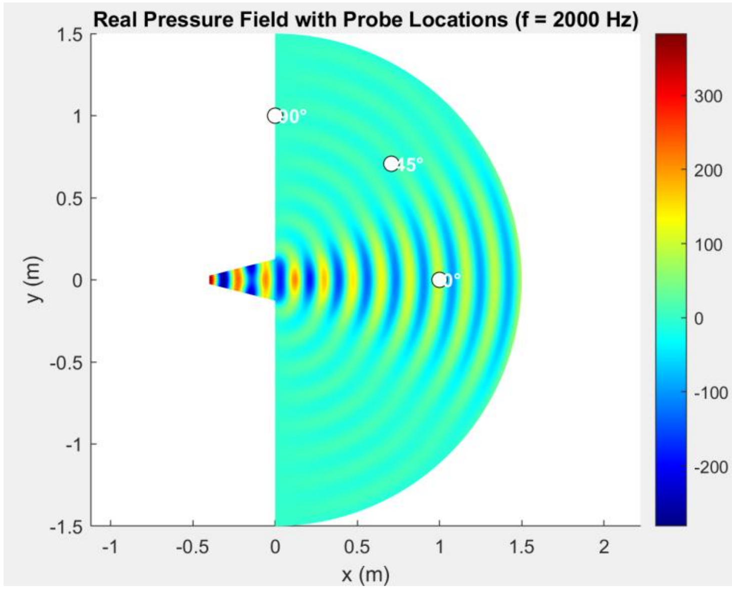
iii. Pressure Field & SPL (SPL = 20*log(p/p_ref) pressure conversion)





2. Finding Pressure and SPL data

a. Command Prompt write with data at 0 (on-axis), 45, 90 degrees




```
Angle:   0° | Pressure: 106.0520 Pa | SPL: 134.49 dB
Angle:  45° | Pressure: 12.4361 Pa | SPL: 115.87 dB
Angle:  90° | Pressure: 1.2402 Pa | SPL: 95.85 dB
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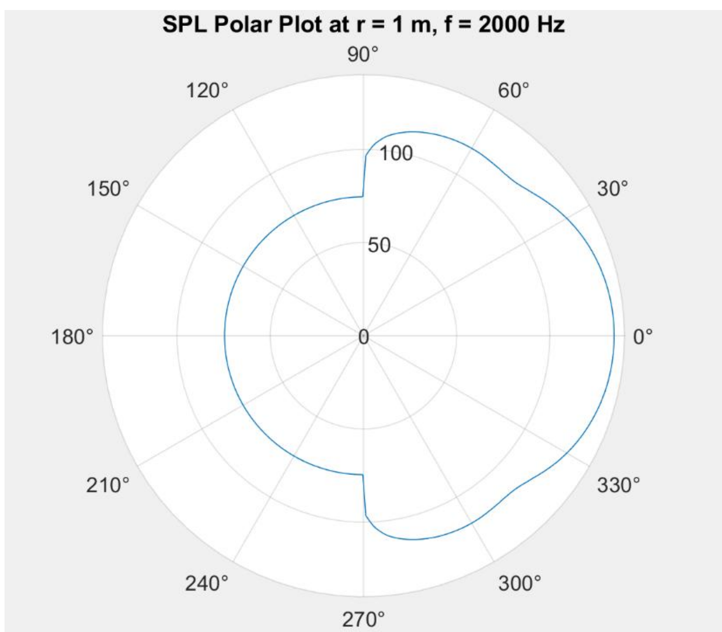
b. 360 Polar Plot (sampling 360 points in circle at 1m, interpolating value in-between)



3. **Lumped Radiation Impedance (avg pressure at throat to the volume velocity flowing through that surface)**
   a. **Finding Lumped Radiation Impedance (Za)**
      i. The average pressure at the throat of the horn is found to be divided by the particle velocity that is multiplied by the throat cross-sectional area.

$$Z_a = \frac{p_{\text{mean}}}{q} = \frac{p_{\text{mean}}}{v \cdot S_{\text{th}}}$$

   b. **Finding Reference Impedance (Zao)**
      i. This is the characteristic acoustic impedance of the throat in the high-frequency limit
      ii. Represents the ideal load for a wave propagation normally through a circular duct

$$Z_{a0} = \frac{\rho c}{S_{\text{th}}}$$

   c. **Finding Normalized Radiation Impedance (Znorm)**
      i. Z_a_norm = Z_a / Z_a0
      ii. Gives unitless complex value
         - Real Values (resistance): part of the wave that radiates and carries power
         - Imaginary Values (reactance): associated with stored (non-radiated energy near the throat)

d. **Sample Results (100Hz to 10kHz sweep, sampling every 100 Hz, took 18 minutes)**